Cardinal Characteristics & Partition Properties

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Abstract

I am going to give an account on connections between the Cardinal Characteristics of the Continuum on the one side and Partition Relations on the other. The $\mid^{\bullet}$-principle, introduced in [978B] allows for a generalisation to a Cardinal Characteristic as done in [997F]. The term $\mid^{\bullet}$ then denotes the minimal size of a family of infinite sets of countable ordinals such that every uncountable set of countable ordinals contains a member of the family. The term $\mathfrak{b}$ denotes the unbounding number and $\mathfrak{d}$ the dominating number. The linear refinement number $\mathfrak{lr}$ is the minimal cardinality of a centered family in $[\omega]^\omega$ that has no linear refinement, c.f.[016M] and it is shown in Theorem 2.2 ibid. that $\mathfrak{lr} = \aleph_1$ implies $\mathfrak{d} = \aleph_1$. We are able to prove the following:

$$\min(\mathfrak{b}, \mid^{\bullet}) = \aleph_1 \implies \omega_1 \not\to (\omega_1, \omega + 2)^2, \tag{1}$$

$$\mathfrak{b} = \mathfrak{d} \implies \binom{\mathfrak{d}}{\omega} \not\to \begin{bmatrix}\mathfrak{b}\\ \omega\end{bmatrix}^{1,1}_{\aleph_0}, \tag{2}$$

$$\mathfrak{lr} = \aleph_1 \implies \binom{\omega_1}{\omega_1} \not\to \begin{bmatrix}\omega_1 & \vee & \omega\\ \omega & & \omega_1\end{bmatrix}^{1,1}_{\aleph_0}, \tag{3}$$

$$\max\left(\mathfrak{b}, \min(\mid^{\bullet}, \mathfrak{d})\right) = \aleph_1 \implies \omega_1\omega \not\to (\omega_1\omega, 3)^2. \tag{4}$$

This is joint work with William Chen continuing earlier research by Garti, Larson, Shelah, Takahashi and Todorcevic and solving some open problems of [016GS].

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